The Ineffectiveness of the Optimal Merger Regulation*

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Abstract

We study the design of horizontal merger regulation in a Cournot competition setting, where firms are privately informed about production technology. More specifically, a consumer-surplus-maximizer regulator designs a mechanism which determines whether the merger is blocked or accepted, and sets structural remedies (divestitures). This problem does not have the usual quasi-linear structure commonly assumed in the mechanism design literature. We first characterize incentive-compatible mechanisms and then find the optimal one. The complete information case is also presented as a benchmark. Asymmetric information induces important distortions in regulatory decisions. First, every rejected merge would improve consumer surplus. Second, every merge that decreases consumer surplus would be approved. Lastly, every merge rightly approved would be asked fewer divestitures than the optimal one (under-fixing effect). These results seem consistent with recent empirical evidence on the ineffectiveness of the merger regulation.

1 Introduction

In a market with few competitors (oligopoly), society's welfare might be primarily affected by a firms merger. Although in most of the countries there is an antitrust authority in charge of either accept or block mergers (and set remedies in some conditionally

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accepted cases), the empirical literature on ex-post mergers evaluations shows that a significant fraction of accepted mergers increased prices, even after some anti-competitive remedies where required.[1] Merger regulation has not been effective, and as a result, the loss in society's welfare is significant.

In this paper, we propose a simple model to understand what is the optimal rule for the regulator to follow. We show that a plausible explanation for the ineffectiveness of merger regulation is the asymmetric information between the regulator and the firms competing in the market. In this context, an optimal regulation not only takes the best decision for the society but also needs to give enough incentives for the firms to report truthfully to the regulator the information that they privately have. Since the last is a necessary condition to make an informed decision, the regulator will make mistakes in the acceptance decision and the number of remedies required.

We use a mechanism design approach. In the first period, the regulator will commit to a merger regulation that condition the decisions to the messages send by the firms. We interpret those messages as the information that firms provides to the regulator in order to persuade it for some decision. In the next period, firms will send messages, and later on, they will compete in a context according to the regulator decision (and possible remedies). We assume a simple competition in quantities (Cournot) and a regulator that follows a consumer surplus standard. From a technical perspective, the structure of the problem will result in firms having non-quasi-linear payoffs, and moreover, depending on all firms information (interdependent value problem).

First, we characterize an incentive compatible merger rule and find that the merger decision (to either block or accept the merger) determines remedies. This property is reminiscent of the revenue equivalence theorem in a quasi-linear payoffs environment, where one mechanism instrument (transfers) is determined by the other (allocation decision). This feature will be useful in order to characterize the set of incentive compatible merger rules. Second, we characterize the optimal merger rule and show its existence. Finally, we compare this rule with the optimal one in the complete information case and show how they differ (distortions). We find that asymmetric information induces important distortions in regulatory decisions. Three critical distortions are identified. First, every rejected merge would improve consumer surplus. Second, every merge that decreases consumer surplus would be approved. Lastly, every merge rightly approved would be asked fewer divestitures than the optimal one (under-fixing effect).

[1] Kwoka (2014)

The article proceeds as follows. We discuss the actual literature in horizontal mergers in Subsection 1.1. We describe our model in Section 2. In Section 3 we derive the optimal merge rule when there are no informational issues between firms and regulator (benchmark). In section 4 we characterize the set of merger rules that are incentive compatible and derive the optimal one. Finally, section 5 concludes.

1.1 Literature Review

The seminal work that studies the profit and welfare effects of horizontal mergers is Farrell and Shapiro (1990a). In a Cournot environment, they showed under what conditions cost improvements are sufficient for a merger to reduce the price. These conditions are relevant for regulators that follow a consumer surplus criterion. They also derive sufficient conditions for the case when the regulator uses an aggregate surplus criterion. The main message is that the production technology of the merger needs to be efficient enough (compared with the individual firms) in order to reduce the price. In related work, Farrell and Shapiro (1990b) assume production technologies that depend on units of capital owned by the firms and showed how a competitive output changes when transfers of capital between firms are considered. This project is based on this model.

A different perspective considers a policy-making approach and uses the enforcement aspects of antitrust. This literature started with Besanko and Spulber (1993). In their model, the regulator cannot observe efficiencies (in the production technology) due to the merger, although the firms integrating the merger can observe this. They showed that it might be better to commit to a consumer surplus criterion even though the true welfare objective is aggregate surplus. The intuition is that after the pre-commitment to a rule that maximizes expected consumer surplus, self-selection by the merging firms increase the average quality of the proposal mergers (profitability is positively correlated with efficiencies that the merger generates), making the regulator more willing to approve mergers in a sequential equilibrium where firms anticipate this fact.

More recent literature in the same spirit of focusing on merger rules is Nocke and Whinston (2013) and Nocke and Whinston (2010). The first one considered a static Cournot setting, when the merger proposed is endogenous (a pivotal firm may choose after bargaining between firms which possible merger to propose). Here the regulator knows everything from the firms but the set of possible mergers from which the pivotal firm picked the finally proposed merger. The regulator can commit ex-ante to its merger rule. They focused on a consumer surplus criterion. They characterized the optimal policy and showed that the last impose a stricter standard on mergers involving larger merger partners (regarding their premerger market share). Specifically, the minimal acceptance increase in consumer surplus is strictly positive for all but the smallest merger partner and is larger the higher is the merger partner's premerger share. The intuition is that the different incentives between pivotal firm and regulator make that proposed mergers to be not necessarily the best for consumers. Thus, the regulator rejects some consumer surplus-improving larger mergers to induce firms to propose instead better smaller ones. In the second one, the authors consider a dynamic environment without informational issues, where the possible mergers do not overlap. The paper showed under which conditions the optimal dynamic policy -that wants to maximize discounted expected consumer surplus- is an entirely myopic policy.

In the same vein, another literature considers transfers of units of capital as remedies for merger policy, but without informational issues. In Vasconcelos (2010) a four-oligopoly model is analyzed, where synergies are possible through the union of units of capital. He assumed a consumer surplus criterion and showed that an over-fixing problem associated with remedial divestitures might emerge. Under this, a firm may abstain from proposing a socially desirable merger, anticipating an over-divestiture to obtain the merger approval.

Finally, there is literature that studies the effect of new firms entering the market. Intuitively, the possibility of post-merger entry reduces the set of profitable mergers. If we are interested in a consumer surplus standard, the possibility of entry of firms increases the likelihood that a given merger will lower the price. In Werden and Froeb (1998) it is showed that mergers that lead firms entry in the future are rarely profitable in the absence of efficiencies. Thus, profitable mergers will be heavily weighted towards mergers that reduce costs. In a recent paper, Pesendorfer (2005), a repeated game with endogenous merger and entry of firms is studied. Two properties are established. First, a merger for monopoly may not be profitable (Because the entrant of new firms and the no future possibility to merge). Second, a merger in a no concentrated industry can be profitable (when future expected mergers exist).

While most of the literature considers environments where the regulator knows the production technology, this project intends to recognize the informational asymmetry that exists between regulator and firms (not only the firms within the merger). Besides that, we consider the possibility for the regulator to set remedies in a similar sense than Farrell and Shapiro (1990b) and Vasconcelos (2010). However, even with a richer set of instruments to regulate we show the ineffectiveness of the optimal rule.

2 Model

Consider a set $I = \{1, \ldots, n\}$ of firms. Each firm is characterized by parameters $(\theta_i, k_i) \in [\underline{\theta}, 1] \times K$, $K \subset \mathbb{R}_+$ and $\underline{\theta} > 0$. Each firm produces one good with constant marginal costs technology, given by $c(\theta_i, k_i) = \theta_i \left(\bar{k} - k_i\right)$, with $\bar{k} \in \mathbb{R}$ a constant sufficiently large compared with k_i[2]. This setup corresponds to a particular case considered in the seminal works Farrell and Shapiro (1990a) and Farrell and Shapiro (1990b) for horizontal mergers, and allow us to capture in the simplest way the dynamics in this problem[3]. We interpret the parameter θ_i as a measure of productivity for a fixed amount of capital (the smaller is this value, the more productive is the firm), and k_i as the units of capital owned by firm i.[4] Denote $(\theta, k) = \left((\theta_1, \ldots, \theta_n), (k_1, \ldots, k_n)\right) \in \Theta \times \mathcal{K}$, where $\Theta \equiv [\underline{\theta}, 1]^n$ and $\mathcal{K} \equiv K^n$. Firms compete in quantities (Cournot competition), with an inverse demand function $P(Q) = 1 - Q$.

Under this setting, there is a "well behaved"[5] Nash equilibrium for any state of the economy (θ, k). The comparative statics for different (θ, k) are well studied in Farrell and Shapiro (1990a).

We think a merger as a change in the state of the economy from (θ, k) to (θ', k'), where the last one depends on the former one in a way to capture the idea of a group of firms merging. From now on, we explicitly write each variable on equilibrium as a function of the state of the economy that produce that output. Denote firm i's payoff as $\pi_i(\theta, k) = \left[P(Q(\theta, k)) - c(\theta_i, k_i)\right] q_i(\theta, k)$ with $Q(\theta, k) = \sum_{i \in I} q_i(\theta, k)$. Note that on equilibrium, all firms parameters affect each firm payoffs.

We say a *merger* is any subset of firms $M \subsetneq I$. We think a merger as the union of all firms in the set M. The new firm parameters, denoted by (θ_M, k_M), depend in the parameters of the firms in M. We assume there is an exogenous merging technology

[2] We assume $\bar{k} \in \left(1, \frac{n}{n-1}\right)$ and $k_i = \bar{k} - 1$, for every $i \in I$. This ensure that for any transfer of capital within firms, the marginal cost is positive.

[3] We rule out scale effects. Here firms want to merge only because synergies that affect marginal costs.

[4] We think a situation where each firm needs units of capital to operate, and the total amount of capital in the economy is limited. That creates an oligopoly environment.

[5] "Well behaved" means that if you have (θ, k) and (θ', k') such that $(\theta_i, k_i) = (\theta'_i, k'_i)$ for each $i \neq j$ and $\theta_j < \theta'_j$ or $k_j > k'_j$, then in (θ', k') compared to (θ, k), both firm j and total production increase.
More generally, this is true for any demand function such that: For any $Q > 0$ such that $P(Q) > 0$, the following three conditions hold (i) $P'(Q) < 0$, (ii) $P'(Q) + QP''(Q) < 0$. and (iii) $\lim_{Q \to \infty} P(Q) = 0$

$\mu : [\underline{\theta}, 1]^{|M|} \to [0, 1]$ that gives the new firm productivity parameter. Thus, (θ_M, k_M) is the result of $\theta_M = \mu(\theta|_M)$ (where $\theta|_M$ is the restriction of θ to the parameters from subset M), the new productivity parameter, and $k_M = \sum_{i \in M} k_i$, the sum of the units of capital from firms in M. Consider WLOG that $M = \{(|I| - |M| + 1), \ldots, |I|\}$, thus $I = \{1, \ldots, (|I| - |M|)\} \cup M$. What a merger creates is a new set of firms $I' = \{1, \ldots, (|I| - |M| + 1)\}$, with vector of parameters (θ', k') such that $(\theta'_i, k'_i) = (\theta_i, k_i)$, every $i \leq (|I| - |M|)$ and $(\theta'_i, k'_i) = (\mu(\theta|_M), \sum_{j \in M} k_j)$ for $i = (|I| - |M| + 1)$.

For a given a set of firms I, their characteristics (θ, k) and a merge M, we are interested in compare two equilibrium scenarios: with and without a merge.

Denote firm i's difference in profit due the merger $\Delta\pi_i(\theta, k)$, with $i \in I'$. We have the following:

$$\Delta\pi_i(\theta, k) = \begin{cases} \pi_M(\theta', k') - \sum_{j \in M} \pi_j(\theta, k) & \text{if } i = M \\ \pi_i(\theta', k') - \pi_i(\theta, k) & \text{if } i \in I \backslash M. \end{cases}$$

Besides the firms, there is an Antitrust Authority (AA) that has the ability to block a merger and/or set remedies (divestitures)[6]. We think these remedies as the AA ability to require transfers of capital from the merger to the rest of firms. Assume that AA objective is to maximize the change in consumer surplus[7] $\Delta CS(\theta, k) = CS(\theta', k') - CS(\theta, k)$.

Regarding information, we assume that the capital vector k is perfectly observable by all the firms and the AA. The only source of incomplete information is the productivity vector θ. Thus, θ_i is private information for each firm i. Assume each θ_i is an independent random variable, with cumulative distribution F_i and density f_i.[8] Additionally, all merger firms can observe others partners parameters. We think θ as a random variable with cumulative distribution function F and density f. The support of θ is $[\underline{\theta}, 1]^{|I|}$. We also assume that $f(\theta) > 0$. The function F encompass all the previous information of the AA about firms productivities.

We impose the following assumption over the merging technology μ:

Assumption: μ is a continuous function such that: $\mu : [\underline{\theta}, 1]^{|M|} \to [0, \underline{\theta}]$

This assumption implies that any merger gives a productivity parameter θ_M smaller

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[6]We focus on structural remedies.

[7]Here $CS(\theta, k) = \int_0^{Q(\theta,k)} P(x)dx - Q(\theta, k)P(Q(\theta, k))$

[8]Denote $f_A = \prod_{i \in A} f_i$, $A \subset I$ and $f = \prod_{i \in I} f_i$. The same for F_A and F.

than any other individual firm parameter. This assumption is important for three reasons. First, we ensure that any divestiture increases consumer surplus. So, we are focusing only on cases when divestitures are useful in increase consumer surplus. Second, it ensures that there exists a merger that is profitable for at least some set of parameters. Lastly, this condition is important for existence purposes of an optimal mechanism.

We think that a necessary condition for a group of firms to propose a merger is to be profitable. This condition leads to the following assumption:

Assumption: Any proposed merger M is profitable, $\Delta\pi_M(\theta, k) \geq 0$

From a theoretical point of view, we follow a mechanism design approach, with the AA designing a rule that elicits the information from the firms, to take an informed decision. Firms care about the difference in profits due to the merge, and the AA cares about the difference in consumer surplus. Contrary to the usual mechanism design problems, here we do not have a quasi-linear structure in firms payoffs. However, the AA can set divestitures from the merger firms to his rivals which can be seen as transfers among firms, but they are not linear in firms payoffs. Additionally, note that the difference in profits depends on all the firms' information; thus this case corresponds to an interdependent value problem. Moreover, since in the case of no-merge each firm gets the profit from the no merge case, we think this problem as one with type-dependent participation. However, the simple structure of the problem allows us to overcome this problem. As usual in mechanism design, we use the revelation principle to focus on direct mechanisms. More formally, we have the following:

Fix a merger M. For simplicity, assume that $k_i = k_j = \bar{k} - 1$, every $i, j \in I$[9], so in the case of no merge the marginal cost of firm i is $c(\theta_i) = \theta_i$.

Definition: A *merger rule* is a pair of functions (x, δ) such that:

$$x : \Theta \to \{0, 1\}$$

$$\delta = (\delta_i)_{i \in I'} \text{ and } \delta_i : \Theta \to \mathbb{R}_+$$

The first function is the *merger decision* of allowing ($x = 1$) or blocking ($x = 0$) the merger. The second one is the *divestiture function* which are the divestitures received by each non-merged firm and the divestitures given by the merger. Note that a merger rule induces a new pair (θ', k') in case of merger: $(\theta'_i, k'_i) = (\theta_i, k_i + \delta_i)$, every $i \leq (|I| - |M|)$

[9]Thus $(\bar{k} - k_i) = 1$, every $i \in I$

and $(\theta'_i, k'_i) = (\mu(\theta|_M), \sum_{j\in M} k_j - \delta_M)$ for $i = (|I| - |M| + 1)$. Note that the vector k' only depend on δ, so from now on we write payoffs as a function only on δ. Firm i's payoffs induced by a merger rule (x, δ) is $x(\hat{\theta})\Delta\pi_i(\theta, \delta(\hat{\theta})) + \pi_i(\theta)$, where $\hat{\theta}$ is a vector of reports, and $\pi_i(\theta)$ is the profits in case of no merger. As usual, we require that all firms to participate in the mechanism. We assume that the outside option from not participate is the no merger case profits, in which case firms get $\pi_i(\theta)$.

For any proposed merger M, denote $\Delta(\theta) = \{\delta \in \mathbb{R}_+^{n-m+1} : \Delta\pi_M(\theta, \delta) = 0, \sum_{i\in I\setminus M} \delta_i = \delta_M\}$, the set of divestitures that make indifferent the merger firms to propose the merger and not do it. Denote $\overline{\delta}(\theta) = \sup\{\delta_M : \delta \in \Delta(\theta)\}$, the maximum amount taken from the merger. Note that there is a natural upper bound in the amount of divestitures taken from the merger $\overline{\delta} = \sup_{\theta\in[\underline{\theta},1]^{|I|}} \overline{\delta}(\theta)$.

Assumption: For any proposed merger M, there exists $\delta \in \Delta(\theta)$ such that $\sum_{i\in I\setminus M} \delta_i = \delta_M$ and $\Delta\pi_i(\theta, \delta) \geq 0$ for $i \in I'$

This assumption says that it is always possible to divide the divestitures taken from the merger between the firms outside the merger such that they are at least indifferent about the merger. Any proposed merger that does not satisfy this condition, would have the AA violating some firm participation constraint.

Using the previous, we have the following definitions:

Definition: A merger rule (x, δ) is *incentive compatible* (IC) if truth telling is an ex post Nash equilibrium; that is, if for all $i \in I'$, $\theta \in \Theta$, and $\hat{\theta}_i \in [\underline{\theta}, 1]$

$$x(\theta)\Delta\pi_i(\theta, \delta(\theta)) \geq x(\hat{\theta}_i, \theta_{-i})\Delta\pi_i(\theta, \delta(\hat{\theta}_i, \theta_{-i}))$$

Note that in this definition we are implicitly assuming that the merger firms report jointly to the AA. A merger report consists on a vector $\hat{\theta}_M$, that gives the productivity parameter for each firm in the merger.

Definition: A merger rule (x, δ) is *individually rational* (IR) if each firm, conditional on his type, is willing to participate; that is, if for all $i \in I'$, $\theta \in \Theta$[10]

$$x(\theta)\Delta\pi_i(\theta, \delta(\theta)) \geq 0$$

[10]Note that the original condition is: $x(\theta)\Delta\pi_i(\theta, \delta(\theta)) + \pi_i(\theta) \geq \pi_i(\theta)$

Definition: A merger rule (x, δ) is *feasible* (F) if the amount of capital taken from the merger is divided exactly to the rest of firms; that is if, for all $i \in I'$, $\theta \in \Theta$

$$\sum_{i \in I \setminus M} \delta_i(\theta) = \delta_M(\theta)$$

The AA problem is to select over all possible IC, IR, and F merger rules, the one that maximizes the expected value of the difference in consumer surplus.

We start the analysis with a subset of firms M that decide to propose a merger. We do not model this explicitly. Instead, we consider this as something exogenous to the model. The timing of the game is the following: First, θ is drawn from F. Then the AA commit to a merger rule (x, δ). This rule has as an input the proposed merger. After observing this rule, each non-merged firm and the merger decide to either participate or not in the mechanism. If all the firms decide to participate, then each non-merged firm and the merger report to the AA and a merger decision and divestitures are implemented. Then, firms compete in quantities. If one of the firms decide not to participate, then there is no merge and firms compete in quantities.

3 Complete Information Case

In this section, we study as a benchmark the case when the AA observe the productivity parameter vector θ. In this case the optimal merger rule decision is easy: Accept the merger whenever the difference in consumer surplus is positive after divestitures. The possibility to require divestitures could potentially help to change the difference in consumer surplus from a negative to a positive value. This is the simple intuition explored in the next results. Formally, the AA problem is the following:

$$\max_{x(\theta), \delta(\theta)} x(\theta) \Delta CS(\theta, \delta(\theta))$$

$$\text{subject to: } (IR), (F)$$

Before showing the formal results, it is useful to see an example that gives some intuition of the model.

Example: Consider 3 firms with productivity parameters $\theta = (\frac{1}{8}, \frac{1}{8}, \frac{2}{8})$. Assume that $\overline{k} = 2$ and each firm has one unit of capital, $k_i = 1$. Thus, the marginal costs are $c =$

$(\frac{1}{8}, \frac{1}{8}, \frac{2}{8})$. A Cournot competition between these firms has output quantities $q = (\frac{2}{8}, \frac{2}{8}, \frac{1}{8})$ with total production $Q = \frac{5}{8}$. Profits are $\pi = ((\frac{2}{8})^2, (\frac{2}{8})^2, (\frac{1}{8})^2)$. There are two things to notice. First, lower marginal cost implies higher quantity on equilibrium. Second, profits are increasing in the produced quantity on equilibrium. Let suppose now firm 1 and 2 merge in one, denoted by M. Assume the merger parameter is $\theta_M = \frac{1}{16}$. In this case the new firm will have two units of capital, while the third firm will have only one. Without divestitures, we have $c' = (0, \frac{1}{4})$, $q' = (\frac{5}{12}, \frac{2}{12})$, $Q' = \frac{7}{12}$ and $\pi' = ((\frac{5}{12})^2, (\frac{2}{12})^2)$. Note that the merger is profitable for the merger firms, $\pi'_M > \pi_1 + \pi_2$, and the firm 3, $\pi'_3 > \pi_3$, but consumer surplus decreases $Q' < Q$. Consider now that the AA require a divestiture of $\delta = \frac{4}{7}$ from the merger to firm 3. If this is the case, the new output will be $c'' = (\frac{1}{56}, \frac{6}{56})$, $q'' = (\frac{20}{56}, \frac{15}{56})$, $Q'' = \frac{5}{8}$ and $\pi'' = ((\frac{20}{56})^2, (\frac{15}{56})^2)$. In this case, the merger is still profitable for the merger firms and the other firm, but consumer surplus does not change $Q'' = Q$. Thus, the AA should approve the merger requiring that amount of divestitures. We say a *merger can be fixed* whenever a merger decrease consumer surplus $Q' < Q$ without divestitures, but there exists a positive δ such that it increases consumer surplus, $Q'' \geq Q$, and it is still profitable for the merger and the firm 3.

Important things to note is that the merger profit decreases in the third case compared with the second one, $\pi''_M < \pi'_M$, and the firm 3 profit increases $\pi''_3 > \pi'_3$. In general, the merger profit is decreasing in divestitures, and the receiver firm's profit is increasing on it. Thus, the optimal merger rule is to require as much divestitures as possible up to the point when the merger firms are indifferent between propose the merger or not.

In a more general case, when there are more than one non-merger firms, although the logic is similar, it has some extra details to consider. Suppose there is one firm that does not receive divestitures. For that firm the marginal cost will not change, thus profits will be affected only through the price. In the cases when consumer surplus increases, the price will decrease, and then that firm's profit will decrease. Thus, participation constraints will make the AA to split the divestitures between the non-merger firms in a way to make all firms at least indifferent to the merger.

Most of the intuition from the previous example is summarized in the following lemma.

Lemma 3.1: Consider a feasible merger rule (x, δ). Then:

(i) Merger's profit is decreasing in any divestiture.

(ii) Non-merged firm's profit is increasing in his own divestitures, but decreasing in others firm's divestitures.

(iii) Consumer surplus is increasing in any divestiture. The increment is higher, the higher is the receiver firm's parameter θ_i.[11]

This last property is the key for the optimal merger rule. The AA requires divestitures to the most unproductive firms, paying attention to the participation constraints of the other non-merged firms.

The analysis of the optimal merger rule can be divided in three. First, for some set of parameters θ, the difference in consumer surplus without any divestiture is positive. Then, the merger should be approved whenever divestitures can ensure the participation of the non-merger firms. Second, for other set of parameters the difference in consumer surplus without divestiture is negative. But, requiring divestitures, we can *fix the merger*, making it at least zero, and at the same time respect participation constraints. Thus, the merger should be approved. Finally, for the rest of parameters, the difference in consumer surplus without divestiture is negative, and even after divestitures it will still be negative. In those cases, the merger must de blocked.

Formally, denote Δ_{cs} as the set of divestitures that increase consumer surplus, $\Delta_{cs} = \{\delta \in \mathbb{R}_+^{n-m} : \Delta CS(\theta, \delta) \geq 0\}$, and Δ_m as the set of divestitures that satisfy participation constraints, $\Delta_m = \{\delta \in \mathbb{R}_+^{n-m} : \Delta\pi_i(\theta, \delta) \geq 0, i \in I'\}$. Denote Φ as the set of states $\Phi = \{\theta \in \Theta : \Delta_{cs} \cap \Delta_m \neq \emptyset\}$. A merger M should be accepted if and only if $\theta \in \Phi$. Since consumer surplus is increasing in divestitures, the AA would take as much divestitures as possible from the merge, until the point firms are indifferent to propose it.

Lemma 3.2: The optimal feasible and individually rational merger rule (x, δ) makes the merger firms indifferent to propose the merger or not, requiring divestitures $\delta(\theta) \in \Delta(\theta)$ whenever it is accepted.

The previous characterize the set of states when a merger can be fixed. Then, we are ready to state the optimal merger policy depending in the number of non-merger firms:

[11] In the non-linear case, $c(\theta_i, k_i) = \frac{\theta_i}{\bar{k}+k_i}$ this is also true until a certain value of divestiture determined by the parameters. But we can always consider functions μ (with enough synergies for the merger) such that the amount of divestitures that makes indifferent the merger is smaller than that value. Thus, the same result applies.

Proposition 3.1: In the case $|M| = n - 1$, the optimal merger rule (x, δ) among feasible and individually rational is:

$$x(\theta) = \begin{cases} 1 & \text{if } \theta \in \Phi \\ 0 & \text{In the other case.} \end{cases}$$
$$\delta(\theta) = \overline{\delta}(\theta)$$

The intuition behind the optimal merger rule in this case can be summarized in the following: Since consumer surplus is increasing in divestitures, and the merger is profitable, the AA should require divestitures to the non-merged firm up to the point the merger firms are indifferent to propose the merger. Then, authorize the merger if and only if the consumer surplus is larger than zero. These mergers can de divided in two groups. In the first one, the difference in consumer surplus is already zero without divestitures. And the second one, the difference in consumer surplus without divestitures is negative, but with divestitures this difference is larger than zero.

For the other case, when there are more than one non-merger firms, we need to consider that any divestiture to firm $i \in I \backslash M$ decreases $\Delta\pi_{-i}$, then some participation constraints may be binding. Thus, it may not be possible to give all the divestitures to the most unproductive firm.

Proposition 3.2: In the case $|M| < n - 1$, the optimal merger rule (x, δ) among feasible and individually rational is:

$$x(\theta) = \begin{cases} 1 & \text{if } \theta \in \Phi \\ 0 & \text{In the other case.} \end{cases}$$
$$\delta_M(\theta) = \overline{\delta}(\theta)$$
$$\delta_i(\theta) = \eta_i(\theta),\, i \in I \backslash M$$

Where $(\eta(\theta))_{i \in I \backslash M}$ is the solution of an optimization problem that determines how to divide $\overline{\delta}(\theta)$ between the non-merged firms. This division satisfies the following property: For every non-merger firm but the most unproductive firm, more productive firms receive more divestitures and obtain no gains from the merger. Formally, if $\theta_j \leq \theta_k$, then $\delta_j(\theta) \geq \delta_j(\theta)$; and $\Delta\pi_j(\theta, \delta) = 0$, for every $j, k \neq h$.

Note that in any of these two cases, we have $\delta_M(\theta) = \overline{\delta}(\theta)$. Thus all the gains from the merger vanishes because of the divestitures required by the AA.

4 Incomplete Information Case

In this section, we study the case when the AA does not know the productivity parameter vector θ. We assume the capital vector k is observable and verifiable. Additionally, to set divestitures to improve consumer surplus, the AA may use them as a screening tool. We follow a mechanism design approach, where the structure of the problem results in payoffs from the mechanism that is not linear in divestitures (non-quasi-linear model), and depend on the information from all the firms (interdependent value model). Using the revelation principle, we only focus on direct mechanisms. Assume each firm privately observes his own parameter θ_i, and the merger firms, since they can share information among them, observe all the merger firms parameters $\theta_M = (\theta_i)_{i\in M}$. The equilibrium notion used is ex-post Nash equilibrium, meaning that given others firms are reporting truthfully, each firm optimal report is to do it truthfully (independently of the beliefs about other firms parameters).[12] In the particular case of the merger firms, we assume they jointly report their types, giving space to coordination among them.

We first characterize an incentive-compatible merger rule, and secondly, solve the AA problem.

4.1 Characterization of Incentive Compatibility

Given a merger rule (x, δ), define the acceptance set $A = \{\theta \in \Theta : x(\theta) = 1\}$, and for every $i \in I'$, the conditional acceptance sets $A_i(\theta_{-i}) = \{\theta_i \in [\underline{\theta}, 1]\colon x(\theta) = 1\}$. Consider the following definition:

Definition: We say a firm i is *decisive* given θ_{-i} if $A_i(\theta_{-i}) \neq \emptyset$, and $[\underline{\theta}, 1]\setminus A_i(\theta_{-i}) \neq \emptyset$.

A firm is decisive whenever it can change the merger decision with his report.

The main idea to characterize the incentive compatibility condition is to consider and avoid the possible deviations individually. The first step is to understand how an incentive compatible divestiture looks like for a fixed decision $x(\theta)$ (possible deviations belong only to the acceptance set). Second, we study how to design the merger decision

[12] In particular, when firms observe all other firms parameters, a truthful report is an equilibrium.

$x(\theta)$ such that together with the divestiture $\delta(\theta)$ obtained in the first step, constitutes an incentive compatible merger rule. Finally, we characterize the incentive-compatible rules.

We first obtain a necessary condition for incentive compatibility whenever $\theta_i \in A_i(\theta_{-i})$.

Proposition 4.1: If a merger rule (x, δ) is *incentive-compatible*, then:

(i) For every $i \in I \backslash M$, $\theta_{-i} \in \Theta_i$ and $\theta_i \in A_i(\theta_{-i})$, $\delta_i(\theta) = \delta_i(\theta_{-i})$; that is, divestitures for firm i do not depend on firm i's report.

(ii) For every $\theta_{-M} \in \Theta_{-M}$ and $\theta_M \in A_M(\theta_{-M})$, $\delta_M(\theta) = \delta_M(\theta_{-M})$; that is, divestitures from the merger do not depend on the merger firms reports.

Any divestiture vector δ that have this feature we say satisfies *own-report independence*.

Note that in any *feasible* merger rule, divestitures from the merger M are divided between the rest of firms $i \in I \backslash M$. A useful implication is the following:

Corollary 4.1: In the case $|M| = n - 1$, if a merger rule (x, δ) is incentive-compatible and feasible, then $\delta(\theta) = \delta$ (constant) for every $\theta \in A$.

This corollary shows how restrictive is to require incentive compatibility. In this case, over the acceptance set, there is no possibility to use divestitures as a screening tool. However, as we will see, the divestiture value affects the set of approved mergers, which is used to screen improving consumer surplus mergers.

The next proposition links the divestitures with the merger decision.

Proposition 4.2: If a merger rule (x, δ) is *incentive-compatible*, then:

(i) For $i \in I'$, if $\Delta\pi_i(\theta, \delta(\theta)) < 0$ and i is decisive given θ_{-i}, then $x(\theta) = 0$.

(ii) For $i \in I'$, if $\Delta\pi_i(\theta, \delta(\theta)) > 0$ and i is decisive given θ_{-i}, then $x(\theta) = 1$.

(iii) $x(\theta)$ is monotone in θ_i (decreasing for $i \in I \backslash M$ and increasing for M).

Intuitively, the merger must give positive gains to all the firms to make them willing to report truthfully. This fact is true for all the firms since the AA needs all firms information to evaluate the merge. A direct corollary is that any incentive-compatible merger rule is individually rational.

A useful implication of the relation between divestitures and the merger decision explored in the previous propositions is that it is enough to know one to determine the other.

Proposition 4.3:

(i) Given a monotone $x(\theta)$ (decreasing for $i \in I \backslash M$ and increasing for M), there is only one $\delta(\theta)$ such that (x, δ) is incentive-compatible.[13]

(ii) Given a vector $\delta(\theta)$ that satisfies own-report independence, there is only one $x(\theta)$ such that (x, δ) is incentive-compatible.

We say x (δ) is *induced* by δ (x) referring to this previous one to one relation . This last feature of incentive compatible merger rule is particularly useful to simplify the AA maximization problem. The fact that divestitures depend uniquely in the merger decision is reminiscent to the usual mechanism design quasi-linear environment, where transfers are determined (up to a constant) by the mechanism allocation (well known as the revenue equivalence theorem). What this proposition suggests is that this feature may not be exclusive from the quasi-linear environment.

Using the previous propositions, we finally state the characterization of incentive compatible merger rules.

Proposition 4.4: A merger rule (x, δ) is incentive compatible if and only if δ satisfies own-report independence and x is the *induced* merger decision by δ.

In an incentive compatible merger rule a non-merger firm (merger) report only change the outcome through the merger decision x, because divestiture to receive (give) is a function of others firms reports. Given a merger decision x, the number of divestitures received by each firm $i \in I \backslash M$ in case of having an approved merger is the amount that makes the merger indifferent to the most unproductive type that will make the merger accepted (*pivotal type*).[14]. This structure is analog to the one observed in the context of object allocation problems when agents have interdependent values in a quasi-linear environment. In that case, any monotone allocation has transfers with similar characteristics. For example, in the generalized VCG transfers (for the interdependent case, see Krishna (2009)), an agent report only affects the object allocation decision, but

[13] This is true up to divestitures over the non-acceptance set, which do not affect firms payoffs.

[14] This type is what we denote in the proofs $\hat{\theta}_i(\theta_{-i})$

the others agents reports determine transfers. Moreover, transfers are such that the last type that receives the object is indifferent between receive it or not (pivotal).

4.2 Regulator Problem

In this subsection, we study the AA problem. We use the incentive compatible characterization to write the AA maximization problem more conveniently way and then solve it. The AA set a merger rule that maximizes the expected difference in consumer surplus (with respect to the unknown vector θ) subject to incentive compatibility and individually rational constraints. We assume the AA can commit to this rule.

The AA problem is the following:

$$\max_{(x,\delta)} \int_{\Theta} x(\theta)\Delta CS(\theta,\delta(\theta))dF(\theta)$$

$$\text{subject to: } (IC), (IR), (F)$$

Note that from the previous discussion, (IC) implies (IR).
Consider first the case $|M| = n-1$. Using Corollary 4.1 and Proposition 4.4, the problem is reduced to find an optimal real value δ^* which is transferred from the merger to the non-merger firm.

Using proposition 4.3, we focus only on the value δ, assuming that the merger decision x is the induced one by δ. The AA problem can be rewritten as follows:

$$\max_{\delta\geq 0} \int_{\Theta} x(\theta)\Delta CS(\theta,\delta)dF(\theta)$$

$$\text{subject to: } x(\theta) \textit{ induced} \text{ by } \delta.$$

Denote $I(\delta)$ the set of θ where all firms get positive gains from the merger, $I(\delta) = \{\theta \in \Theta : \Delta\pi_i(\theta,\delta) \geq 0, i \in I'\}$. Over this set the mergers are all accepted. By proposition 4.2, the problem becomes:

$$\max_{\delta\geq 0} \int_{\Theta} \Delta CS(\theta,\delta)\mathbb{1}_{I(\delta)}(\theta)dF(\theta)$$

The next proposition is a direct consequence of the previous steps.

Proposition 4.5: In the case $|M| = n - 1$, the optimal merger rule (x^*, δ^*) among feasible, individually rational and incentive compatible rules is:

$$x^*(\theta) = \begin{cases} 1 & \text{if } \theta \in I(\delta^*) \\ 0 & \text{In the other case.} \end{cases}$$

$$\delta^* = arg \max_{\delta \geq 0} \int_{\Theta} \Delta CS(\theta, \delta) \mathbb{1}_{I(\delta)}(\theta) dF(\theta)$$

The optimal merger rule requires divestitures that maximize the expected value of a function that is equal to the difference in consumer surplus when all firms receive positive gains from the merger and zero in the other case. Given the optimal divestiture, the merger decision is to accept whenever all firms receive positive gains from the merger. Note that requiring an extra unit of divestiture increases consumer surplus of approved mergers, but reduces the set of parameters θ (inclusion sense) where the merger is accepted because it reduces the set of parameters where all the firms receive positive gains. Thus, the optimal divestiture balances an increase in the difference in consumer surplus in accepted mergers, with the reduction in accepted mergers.

Consider now the case $|M| < n - 1$. Define analogously $I(\delta) = \{\theta \in \Theta : \Delta\pi_i(\theta, \delta(\theta)) \geq 0, i \in I'\}$, but now as a function of a feasible vector δ. We can apply the same logic than before and reduce the AA problem to find $(|M| - n)$ functions $\delta_i(\theta_{-i-M}) : [\underline{\theta}, 1]^{N-M-1} \to \mathbb{R}_+$, $i \in I \backslash M$. The problem is reduced to the following:

$$\max_{(\delta_i(\theta_{-i-M}))_{i \in I \backslash M}} \int_{\Theta} \Delta CS(\theta, \delta(\theta)) \mathbb{1}_{I(\delta)}(\theta) dF(\theta)$$

We have the following result:

Proposition 4.6: In the case $|M| < n - 1$, the optimal merger rule (x^*, δ^*) among feasible, individually rational and incentive compatible rules is:

$$x^*(\theta) = \begin{cases} 1 & \text{if } \theta \in I(\delta^*) \\ 0 & \text{In the other case.} \end{cases}$$

$$\delta_i^*(\theta_{-i-M}) = arg \max_{(\delta_i(\theta_{-i-M}))_{i \in I \backslash M}} \int_{\Theta_i} \int_{\Theta_M} \Delta CS(\theta, \delta(\theta)) \mathbb{1}_{I(\delta)}(\theta) dF_M(\theta_M) dF_i(\theta_i), \; i \in I \backslash M$$

Note that divestitures are characterized by $(|M| - n)$ maximization problems.

4.3 Discussion

In this section, we compare the complete and incomplete information optimal merger rules. For tractability, we only consider the case $|M| = n - 1$.

Proposition 4.7: In the case $|M| = n - 1$, the optimal merger rule in the incomplete information case satisfies the following:

(i) Every rejected merge improve consumer surplus.

(ii) Every merge that decrease consumer surplus is approved.

(iii) Every approved merge is asked less divestitures than the complete information case (under-fixing effect).

From this proposition, we can distinguish three different distortions that result from informational issues. First, all the mergers rejected by the AA improve consumer surplus. Intuitively, since the AA must reject all the mergers where some firm incurs in a decrease in profit due to the merge, this happens when the price on equilibrium in the merger case is relatively low in comparison to the price in the no merger case. Those cases have in common that consumer surplus increases. Second, the AA approves all the mergers that give positive gains to all the firms. Among the mergers that satisfies this conditions, we have all the mergers where the price on equilibrium in the merger case is relatively high in comparison to the price in the no merger case. Then, all these mergers are approved. Lastly, there are approved mergers that effectively increase consumer surplus. However, the AA requires less than the complete information case in order to have more mergers accepted.

5 Conclusion

In this paper, we study the design of horizontal merger regulation when there is asymmetric information related with the production technology between regulator and firms. We characterize incentive compatible mechanisms and then find the optimal one. We also study the complete information case as a benchmark. We show that asymmetric information induces the following distortions: First, every rejected merge would improve consumer surplus. Second, every merge that decreases consumer surplus would be approved. Lastly, every merge rightly approved would be asked fewer divestitures than the optimal one (under-fixing effect). These results give an informational explanation to a significant fraction of accepted mergers that had increased prices even after the requirement of divestitures. In light of these results, it would be interesting to study new tools or methodologies for antitrust authorities to use that may help to reduce these distortions. We leave this for future research.

A Proofs of Section 3

Lemma 3.1: Consider a feasible merger rule (x, δ). Then:

(i) Merger's profit is decreasing in any divestiture.

(ii) Non-merged firm's profit is increasing in his own divestitures, but decreasing in others firm's divestitures.

(iii) Consumer surplus is increasing in any divestiture. The increment is higher, the higher is the receiver firm's parameter θ_i.

Proof

(i) Denote c_i the marginal cost of firm i. The first order conditions that determine the competition outcome are the following:

$$P(Q) - q_i = c_i, \text{ for every } i \in I$$

Summing over firms in I:

$$NP(Q) - Q = \sum_{i \in I} c_i$$

Then the total output is:

$$Q = \frac{N - \sum_{i \in I} c_i}{N+1}$$

Firm i production is $q_i = 1 - Q - c_i$ and profit $\pi_i = q_i^2$. In the merger case, considering divestitures, we have that $c_M = \theta_M(\bar{k} - M(\bar{k} - 1) + \delta_M)$ and $c_i = \theta_i(1 - \delta_i)$, with $i \in M \backslash I$. Note that firm i profit depend negatively on $Q + c_i = \frac{N - \sum_{j \neq i} c_j + Nc_i}{N+1}$. Thus, in the merger case,

$$Q + c_M = \frac{(N - M + 1) - \sum_{j \neq i} \theta_j (1 - \delta_j) + (N - M + 1)\theta_M(\bar{k} - M(\bar{k} - 1) + \delta_M)}{(N - M + 1) + 1}$$

Which is increasing in δ_M and δ_i. Thus merger profit π_M is decreasing in δ_M and δ_i and then decreasing in any divestiture.

(ii) By the same argument we have that for $i \in M \backslash I$:

$$Q + c_i = \frac{(N - M + 1) - \sum_{j \neq i, M} \theta_j (1 - \delta_j) - \theta_M(\bar{k} - M(\bar{k} - 1) + \delta_M) + (N - M + 1)\theta_i(1 - \delta_i)}{(N - M + 1) + 1}$$

This is decreasing in δ_i and δ_M, and increasing in δ_j with $j \neq i, M$. Thus firm i profit π_i is increasing in δ_i and δ_M, and decreasing in δ_j with $j \neq i, M$. Then profits are increasing in any divestiture to firm i. Moreover, since $\theta_M \leq \theta_j$, then it is decreasing in any divestiture to firm j.

(iii) From previous calculation we have that:

$$Q = \frac{(N-M+1) - \sum_{i \in I} c_i}{(N-M+1)+1} = \frac{(N-M+1) - \sum_{i \neq M} \theta_i (1-\delta_i) - \theta_M(\bar{k} - M(\bar{k}-1) + \delta_M)}{(N-M+1)+1}$$

This is decreasing in δ_M and increasing in δ_i. Since $\theta_M \leq \theta_i$, then it is increasing in any divestiture.

Lemma 3.2: The optimal feasible and individually rational merger rule (x, δ) makes the merger firms indifferent to propose the merger or not, requiring divestitures $\delta(\theta) \in \Delta(\theta)$ whenever it is accepted.

Proof Consider an accepted merger. By contradiction, suppose $\delta \notin \Delta(\theta)$. Then, $\Delta\pi_M(\theta, \delta) > 0$. Using the continuity of $\Delta\pi$ at δ, there exists δ' such that (x, δ') is feasible and individually rational, with $\delta'_M > \delta_M$. Then $\Delta CS(\theta, \delta') > \Delta CS(\theta, \delta)$ contradicting the optimality of δ. Thus, $\delta \in \Delta(\theta)$.

Proposition 3.1: In the case $|M| = n - 1$, the optimal merger rule (x, δ) among feasible and individually rational is:

$$x(\theta) = \begin{cases} 1 & \text{if } \theta \in \Phi \\ 0 & \text{In the other case.} \end{cases}$$
$$\delta(\theta) = \bar{\delta}(\theta)$$

Proof We have to show that the previous is a solution of the problem:

$$\max_{x(\theta), \delta(\theta)} x(\theta) \Delta CS(\theta, \delta(\theta))$$

$$\text{subject to: } x(\theta)\Delta\pi_i(\theta,\delta(\theta)) \geq 0,\, i \in I' \text{ and (F)}$$

From the previous lemma, whenever $x(\theta) = 1$, $\delta \in \Delta(\theta)$. In the case $|M| = n-1$, $\Delta(\theta) = \{\overline{\delta}(\theta)\}$ by definition of $\overline{\delta}(\theta)$. Then $\delta(\theta) = \overline{\delta}(\theta)$. By definition, $\Phi = \{\theta \in \Theta : \Delta_{cs} \cup \Delta_m \neq \emptyset\}$, thus if $\theta \in \Phi$, there exists δ' such that $\Delta CS(\theta,\delta') \geq 0$ and satisfies feasible and participation constraints. But since $|M| = n-1$, then $\Delta CS(\theta,\delta) \geq \Delta CS(\theta,\delta')$, which still satisfies feasible and participation constraints. Thus $x(\theta) = 1$. Consider now $\theta \notin \Phi$. We know that for every δ, either $\Delta\pi_i(\theta,\delta) < 0$ for some $i' \in I'$ or $\Delta CS(\theta,\delta) < 0$. In the first case, there is no individually rational merger rule, and in the second one, it must be that $x(\theta) = 0$. In any case, $x(\theta) = 0$.

Proposition 3.2: In the case $|M| < n-1$, the optimal merger rule (x,δ) among feasible and individually rational is:

$$x(\theta) = \begin{cases} 1 & \text{if } \theta \in \Phi \\ 0 & \text{In the other case.} \end{cases}$$

$$\delta_M(\theta) = \overline{\delta}(\theta)$$

$$\delta_i(\theta) = \eta_i(\theta),\, i \in I \backslash M$$

Where $(\eta(\theta))_{i \in I\backslash M}$ is the solution of an optimization problem that determines how to divide $\overline{\delta}(\theta)$ between the non-merged firms. This division satisfies the following property: For every non-merger firm but the most unproductive firm, more productive firms receive more divestitures and obtain no gains from the merger. Formally, if $\theta_j \leq \theta_k$, then $\delta_j(\theta) \geq \delta_j(\theta)$; and $\Delta\pi_j(\theta,\delta) = 0$, for every $j,k \neq h$.

Proof We have to show that the previous is a solution of the problem:

$$\max_{x(\theta),\delta(\theta)} x(\theta)\Delta CS(\theta,\delta(\theta))$$

$$\text{subject to: } x(\theta)\Delta\pi_i(\theta,\delta(\theta)) \geq 0,\, i \in I' \text{ and (F)}$$

From the previous lemma, whenever $x(\theta) = 1$, $\delta \in \Delta(\theta)$. In the case $|M| < n-1$, we know that $\delta_M(\theta) = \overline{\delta}(\theta)$ and $\sum_{i \in I\backslash M} \delta_i = \overline{\delta}(\theta)$. By definition, $\Phi = \{\theta \in \Theta : \Delta_{cs} \cup \Delta_m \neq \emptyset\}$, thus if $\theta \in \Phi$, there exists δ' such that $\Delta CS(\theta,\delta') \geq 0$ and satisfies feasibility and participation constraints. But then, in the optimum we now that $\Delta CS(\theta,\delta) \geq \Delta CS(\theta,\delta') \geq 0$, and still satisfies feasible and participation constraints. Thus $x(\theta) = 1$. Consider now $\theta \notin \Phi$. We know that for every δ, either $\Delta\pi_i(\theta, \delta) < 0$ for some $i' \in I'$ or $\Delta CS(\theta, \delta) < 0$. In the first case, there is no individually rational merger rule, and in the second one, it must be that $x(\theta) = 0$. In any case, $x(\theta) = 0$. A feasible divestitures consists on $I \backslash M$ functions $\delta_i : \Theta \to \mathbb{R}_+$ such that $\sum_{i \in I \backslash M} \delta_i(\theta) = \overline{\delta}(\theta)$. The problem of how to divide $\overline{\delta}(\theta)$ units of divestitures among the non-merger firms is the following:

$$Max_{(\delta_i(\theta))_{i \in I \backslash M}} \Delta CS(\theta, \delta(\theta))$$

subject to:

$$\Delta\pi_i(\theta, \delta(\theta)) \geq 0 \text{ , } i \in I \backslash M$$

$$\sum_{i \in I \backslash M} \delta_i(\theta) = \overline{\delta}(\theta)$$

$$\delta_i(\theta) \geq 0 \text{ , } i \in I \backslash M$$

Denote h the firm with the highest parameter θ_i. In the optimum, the restrictions $\Delta\pi_i(\theta, \delta(\theta)) \geq 0$, $i \in I \backslash M, i \neq h$ are binding. Suppose not. Then, there exists j such that $\Delta\pi_j(\theta, \delta(\theta)) > 0$. By continuity, there exist an $\varepsilon > 0$ such that $\Delta\pi_i(\theta, \delta') > 0$ for $i \in M \backslash I$, with $\delta'_i = \delta_i(\theta)$ for $i \in M \backslash I$, $i \neq j$ and $\delta'_j = \delta_j(\theta) - \varepsilon$. Since now all the constraints are not binding, using a continuity argument, we can give some fraction of ε to firm h, and strictly increase consumer surplus while the constrains are still satisfied. This contradicts that some constraint for $i \neq h$ is not binding.

Then the problem can be rewritten a follows:

$$Max_{(\delta_i(\theta))_{i \in I \backslash M}} \Delta CS(\theta, \delta(\theta))$$

subject to:

$$\theta_i \delta_i(\theta) = \Delta Q(\theta, \delta(\theta)), \text{ for } i \neq h$$

$$\theta_h \delta_h(\theta) \geq \Delta Q(\theta, \delta(\theta))$$

$$\sum_{i \in I \backslash M} \delta_i(\theta) = \overline{\delta}(\theta)$$

$$\delta_i(\theta) \geq 0 \text{ , } i \in I \backslash M$$

Denote the solution of this problem $(\eta_i)_{i \in I \backslash M}$. Note that $\theta_j \eta_j = \theta_k \eta_k = \Delta Q(\theta, \eta)$, every $j, k \neq h$. From this we have that if $\theta_j \leq \theta_k$, then $\eta_j \geq \eta_j$ every $j, k \neq h$.

B Proofs of Section 4

Proposition 4.1: If a merger rule (x, δ) is incentive-compatible, then:

(i) For every $i \in I \backslash M$, $\theta_{-i} \in \Theta_i$ and $\theta_i \in A_i(\theta_{-i})$, $\delta_i(\theta) = \delta_i(\theta_{-i})$; that is, divestitures for firm i do not depend on firm i's report.

(ii) For every $\theta_{-M} \in \Theta_{-M}$ and $\theta_M \in A_M(\theta_{-M})$, $\delta_M(\theta) = \delta_M(\theta_{-M})$; that is, divestitures from the merger do not depend on the merger firms reports.

Proof

(i) Consider $i \in I \backslash M$. Fix θ_{-i} and consider a type θ_i. The proof is divided in three steps. In step 1, we show there exist a region $R \subset [\underline{\theta}, 1] \times \mathbb{R}_+$ such that $f(\theta_i) = \delta(\theta_i, \theta_{-i}) : [\underline{\theta}, 1] \to \mathbb{R}_+$ must be no decreasing in R and no increasing in $R^c = [\underline{\theta}, 1] \times \mathbb{R}_+ \backslash R$ (and thus, almost everywhere differentiable). In step 2, we show that the function $f(\theta_i)$ is either increasing or constant function. Finally in step 3 we show that if $f(\theta_i)$ is increasing, it must belong to R^c, having a contradiction. Thus, $f(\theta_i)$ must be constant.

Step 1. Suppose first i is decisive. Consider type $\theta_i \in A(\theta_{-i})$. Any report $\theta_i' \in [\underline{\theta}, 1] \backslash A(\theta_{-i})$ gives him zero payoff. In the other side any report $\theta_i' \in A(\theta_{-i})$ must satify the following condition:

$$\Delta\pi_i(\theta, \delta(\theta)) \geq \Delta\pi_i(\theta, \delta(\theta_i', \theta_{-i}))$$

We can rewrite it only in terms of profit in the merger case.

$$\pi_i(\theta, \delta(\theta)) \geq \pi_i(\theta, \delta(\theta_i', \theta_{-i}))$$

We can rewrite this condition as follows:

$$\pi_i(\theta, \delta(\theta)) = \max_{\theta_i' \in A_i(\theta_{-i})} \pi_i(\theta, \delta(\theta_i', \theta_{-i}))$$

Using envelope theorem we have that:

$$\pi_i(\theta, \delta(\theta)) = \pi_i(\theta_i', \theta_{-i}, \delta(\theta', \theta_{-i})) + \int_{\theta_i'}^{\theta_i} \pi_{i,(1)}(\tilde{\theta}_i, \theta_{-i}, \delta(\tilde{\theta}_i, \theta_{-i})) d\tilde{\theta}_i$$

Where $\pi_{i,(1)}(\theta_i, \theta_{-i}, \delta) = \frac{d\pi_i}{d\theta_i}(\theta_i, \theta_{-i}, \delta)$. Replacing this in the IC condition:

$$\pi_i(\theta_i', \theta_{-i}, \delta(\theta', \theta_{-i})) + \int_{\theta_i'}^{\theta_i} \pi_{i,(1)}(\tilde{\theta}_i, \theta_{-i}, \delta(\tilde{\theta}_i, \theta_{-i}))d\tilde{\theta}_i \geq \pi_i(\theta, \delta(\theta_i', \theta_{-i}))$$

Rearranging terms, we have that this is equivalent to:

$$\int_{\theta_i'}^{\theta_i} \int_{\delta(\theta_i', \theta_{-i})}^{\delta(\tilde{\theta}_i, \theta_{-i})} \pi_{i,(1,2)}(\tilde{\theta}_i, \theta_{-i}, \tilde{\delta})d\tilde{\delta}d\tilde{\theta}_i \geq 0$$

Where $\pi_{i,(1,2)}(\theta_i, \theta_{-i}, \delta) = \frac{d^2\pi_i}{d\delta d\theta_i}(\theta_i, \theta_{-i}, \delta)$. Thus, $\delta(\theta_i, \theta_{-i})$ must be no decreasing in θ_i whenever $\pi_{i,(1,2)}(\theta_i, \theta_{-i}, \delta) \geq 0$ and no increasing whenever $\pi_{i,(1,2)}(\theta_i, \theta_{-i}, \delta) \leq 0$. Note that given this particular environment, $\pi_{i,(1,2)}$ can take both signs for a particular (θ_i, δ). In other words, the usual single crossing property between type and the decision does not hold. This suggest that different shapes for the function $f(\theta_i) = \delta(\theta_i, \theta_{-i})$ may satisfy the above condition (in Araujo and Moreira (2010) a particular shape is discussed). Denote

$$R = \{(\theta_i, \delta) \in [\underline{\theta}, 1] \times \mathbb{R}\text{: such that } \pi_{i,(1,2)}(\theta_i, \theta_{-i}, \delta) > 0\}$$

and

$$R^c = \{(\theta_i, \delta) \in [\underline{\theta}, 1] \times \mathbb{R}\text{: such that } \pi_{i,(1,2)}(\theta_i, \theta_{-i}, \delta) < 0\}$$

Step 2. Define $\delta^*(\theta_i, \theta_{-i})$ as the value δ^* such that $\pi_{i,(1,2)}(\theta_i, \theta_{-i}, \delta^*) = 0$.

Let consider now first order conditions. The IC problem can be rewritten as:

$$\theta_i = arg \max_{\theta_i' \in A_i(\theta_{-i})} \Delta\pi_i(\theta, \delta(\theta_i', \theta_{-i}))$$

For θ_i' in the interior of $A_i(\theta_{-i})$ we must have that:

$$\left. \frac{\partial \Delta\pi_i(\theta_i, \theta_{-i}, \delta)}{\partial \delta} \frac{d\delta(\theta_i', \theta_{-i})}{d\theta_i'} \right|_{\theta_i' = \theta_i} = 0$$

Denote $\delta'(\theta)$ as the function that satisfies the condition before. Thus either $\delta'(\theta) = \delta'(\theta_{-i})$ or $\delta'(\theta)$ solves $\left. \frac{\partial \Delta\pi_i(\theta_i, \theta_{-i}, \delta(\theta_i', \theta_{-i}))}{\partial \delta} \right|_{\theta_i' = \theta_i} = 0$. In the last case, for this particular environment, $\delta'(\theta_i, \theta_{-i})$ is increasing in θ_i.

Step 3. It is direct to check that the last increasing function $\delta'(\theta_i, \theta_{-i})$ belongs entirely to R^c which is a contradiction with the previous conditions. Thus, it must be that $\delta'(\theta) = \delta'(\theta_{-i})$.

(ii) An analogue analysis than part (i) can be made for an individual firm from the merger $i \in M$, resulting in that $\delta_M(\theta) = \delta_M(\theta_{-i})$. This rules out cases of individual misreport from the firm i. Thus, the requirement to $\delta_M(\theta)$ to rule out any individual misreport is to have $\delta_M(\theta) = \delta_M(\theta_{-M})$. But, this also rules out any jointly misreport of types, since the decision of transfer does not depend in any report from the merger firms θ_M. Thus, it must be that $\delta_M(\theta) = \delta_M(\theta_{-M})$.

Proposition 4.2: If a merger rule (x, δ) is incentive-compatible, then:

(i) For $i \in I'$, if $\Delta\pi_i(\theta, \delta(\theta)) < 0$ and i is decisive given θ_{-i}, then $x(\theta) = 0$.

(ii) For $i \in I'$, if $\Delta\pi_i(\theta, \delta(\theta)) > 0$ and i is decisive given θ_{-i}, then $x(\theta) = 1$.

(iii) $x(\theta)$ is monotone in θ_i (decreasing for $i \in I \backslash M$ and increasing for M).

Proof

(i) Consider type θ_i such that $\Delta\pi_i(\theta_i, \theta_{-i}, \delta(\theta_{-i})) < 0$. Suppose by contradiction that $x(\theta_i, \theta_{-i}) = 1$. Since $[\underline{\theta}, 1] \backslash A(\theta_{-i}) \neq \emptyset$, reporting any $\theta_i' \in [\underline{\theta}, 1] \backslash A(\theta_{-i})$ gives a strictly higher payoff, which contradicts IC condition. Thus it must be that $x(\theta_i, \theta_{-i}) = 0$.

(ii) Consider type θ_i such that $\Delta\pi_i(\theta_i, \theta_{-i}, \delta(\theta_i, \theta_{-i})) > 0$. Suppose by contradiction that $x(\theta_i, \theta_{-i}) = 0$. Thus, firm i get zero payoff. Pick $\theta_i' \in A(\theta_{-i})$. Using previous propositions, it must be that $\Delta\pi_i(\theta_i', \theta_{-i}, \delta(\theta_{-i})) = \Delta\pi_i(\theta, \delta(\theta_{-i})) > 0$. Thus type θ_i can report θ_i' and get a strictly positive payoff, which contradicts IC ondition. Thus it must be that $x(\theta_i, \theta_{-i}) = 1$.

(iii) Since $\Delta\pi_i(\theta, \delta(\theta_{-i}))$ is monotone in θ_i, using (i) and (ii) we obtain the monotonicity in θ_i in each case.

Proposition 4.3:

(i) Given a monotone $x(\theta)$ (decreasing for $i \in I \backslash M$ and increasing for M), there is only one $\delta(\theta)$ such that (x, δ) is incentive-compatible.

(ii) Given a vector $\delta(\theta)$ that satisfies own-report independence, there is only one $x(\theta)$ such that (x, δ) is incentive-compatible.

Proof

(i) Consider any monotone $x(\theta)$. Pick any $i \in I \backslash M$. For any θ_{-i}, there is a cut off type $\hat{\theta}_i(\theta_{-i})$ such that $x(\theta) = 1$ if and only if $\theta_i \leq \hat{\theta}_i(\theta_{-i})$. We know that any IC rule must have $\delta_i(\theta_{-i})$. Define $\delta_i(\theta_{-i})$ as the value δ_i that solve the following equation $\Delta\pi_i(\hat{\theta}_i(\theta_{-i}), \theta_{-i}, \delta_i) = 0$. Since δ_i does not depend on θ_i there is no possibility to manipulate. Moreover, it is the case that $\Delta\pi_i(\theta_i, \theta_{-i}, \delta_i(\theta_{-i})) \geq 0$ if and only if $x(\theta) = 1$. Thus, there is no incentives to misreport. Suppose that we define $\delta_i(\theta_{-i}) > \delta_i$. Thus a type $\hat{\theta}_i(\theta_{-i}) - \epsilon$ will report $\hat{\theta}_i(\theta_{-i}) + \epsilon$, with ϵ small enough, and get positive payoffs instead of zero. The case when $\delta_i(\theta_{-i}) < \delta_i$ is analogue. Thus, we have a unique $\delta_i(\theta_{-i})$ such that (x, δ) is IC for agent i. Repeating the argument for all the agents we obtain the result.

(ii) Consider a vector $\delta(\theta)$ that satisfies own-report independence. Thus $\delta_i(\theta) = \delta(\theta_{-i})$. For every i, define define $\hat{\theta}_i(\theta_{-i})$ as the value $\hat{\theta}_i$ that solve the following equation $\Delta\pi_i(\hat{\theta}_i, \theta_{-i}, \delta_i(\theta_{-i})) = 0$. Set $x(\theta) = 1$ whenever $\theta_i \leq \hat{\theta}_i$ with $i \in I \backslash M$, and $\theta_i \geq \hat{\theta}_i$ with $i \in M$. Similar than the case before, since $\delta_i(\theta_{-i})$ does not depend on θ_i there is no possibility to manipulate. Moreover, it is the case that $\Delta\pi_i(\theta_i, \theta_{-i}, \delta_i(\theta_{-i})) \geq 0$ if and only if $x(\theta) = 1$. Thus there is no space to misreport. Thus (x, δ) is IC. Let suppose another x' with the same δ as a merger rule. There must exist some θ'_i such that either $\Delta\pi_i(\theta'_i, \theta_{-i}, \delta_i(\theta_{-i})) > 0$ whenever $x(\theta) = 0$, or $\Delta\pi_i(\theta'_i, \theta_{-i}, \delta_i(\theta_{-i})) < 0$ whenever $x(\theta) = 1$. In both cases, there are incentives to misreport, thus (x', δ) is not IC and the only IC is the (x, δ) with x defined before.

Proposition 4.4: A merger rule (x, δ) is incentive compatible if and only if δ satisfies own-report independence and x is the induced merger decision by δ.

Proof First, suppose (x, δ) is IC. From the previous propositions δ satisfies own-report independence. Since that, given δ there is only one x such that (x, δ) is IC, it must be the induced one defined before. In the other way, consider any merger rule (x, δ) such that δ satisfies own-report independence and x is the induced merger decision by δ. Since δ satisfies own-report independence, there is no possible deviation that change the amount of transfer received (given). Moreover, since x is induced by δ, $x(\theta) = 1$ if and only if $\Delta\pi_i(\theta, \delta(\theta)) \geq 0$. Thus there is no incentives to misreport from the merger decision.

Proposition 4.5: In the case $|M| = n - 1$, the optimal merger rule (x^*, δ^*) among feasible, individually rational and incentive compatible rules is:

$$x^*(\theta) = \begin{cases} 1 & \text{if } \theta \in I(\delta^*) \\ 0 & \text{In the other case.} \end{cases}$$

$$\delta^* = arg \max_{\delta \geq 0} \int\limits_{\Theta} \Delta CS(\theta, \delta) \mathbb{1}_{I(\delta)}(\theta) dF(\theta)$$

Proof Direct from previous propositions.

Proposition 4.6: In the case $|M| < n - 1$, the optimal merger rule (x^*, δ^*) among feasible, individually rational and incentive compatible rules is:

$$x^*(\theta) = \begin{cases} 1 & \text{if } \theta \in I(\delta^*) \\ 0 & \text{In the other case.} \end{cases}$$

$$\delta_i^*(\theta_{-i-M}) = arg \max_{(\delta_i(\theta_{-i-M}))_{i \in I \setminus M}} \int\limits_{\Theta_i} \int\limits_{\Theta_M} \Delta CS(\theta, \delta(\theta)) \mathbb{1}_{I(\delta)}(\theta) dF_M(\theta_M) dF_i(\theta_i), \, i \in I \setminus M$$

Proof Direct from previous propositions.

Proposition 4.7: In the case $|M| = n-1$, the optimal merger rule in the incomplete information case satisfies the following:

(i) Every rejected merge improve consumer surplus.

(ii) Every merge that decrease consumer surplus is approved.

(iii) Every approved merge is asked less divestitures than the complete information case (under-fixing effect).

Proof

(i) Consider θ such that $x(\theta) = 0$. By definition, there is a non-merged firm, which we denote by z, such that either $\Delta\pi_M(\theta, \delta(\theta)) < 0$ or $\Delta\pi_z(\theta, \delta(\theta)) < 0$. If $\Delta\pi_M(\theta, \delta(\theta)) < 0$, that would mean that $\delta(\theta) > \overline{\delta}(\theta)$. Thus, AA requires to much divestitures (over-fixing problem) and the merger is not proposed at the beginning. In the other case $\Delta\pi_z(\theta, \delta(\theta)) < 0$. This is equivalent to $\theta_z\delta_z(\theta) < \Delta Q(\theta, \delta(\theta))$. Since $\theta_z\delta_z(\theta) \geq 0$, then $\Delta Q(\theta, \delta(\theta)) > 0$ which is equivalent to $\Delta CS(\theta, \delta(\theta)) > 0$.

(ii) Consider a merge M with divestitures δ that decrease consumer surplus. Thus $\Delta Q(\theta, \delta) < 0$. Suppose this merger is proposed. Consider a non-merger firm $i \in \backslash M$. $\Delta\pi_z(\theta, \delta) \geq 0$ is equivalent to $\theta_i\delta_i \geq \Delta Q(\theta, \delta)$. Since $\theta_i\delta_i \geq 0$, this is trivially satisfied. Thus $\Delta\pi_z(\theta, \delta) \geq 0$. Since $\Delta\pi_i(\theta, \delta) \geq 0$, for $i \in I'$, then $x(\theta) = 1$.

(iii) Consider θ such that $x(\theta) = 1$ By definition $\Delta\pi_M(\theta, \delta(\theta)) \geq 0$. In the complete information case, any approved require divestiture $\overline{\delta}(\theta)$. Since $\Delta\pi_M(\theta, \delta) \geq 0$ is decreasing in δ, we conclude that $\delta(\theta) \leq \overline{\delta}(\theta)$.

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