Competing Auctions with Informed Sellers

Zizhen Ma   Nicolas Riquelme

Department of Economics
University of Rochester

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Introduction
Motivation

Auctions are widely used to allocate objects among agents with unknown willingness to pay

- Housing market
- Procurement
- Ebay

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This paper:

- How sellers choose term of trade?
- How buyers choose which auction to participate?
- Efficiency properties

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Introduction
Literature

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Together, sellers have opposite incentives

Questions:

- Can a seller still signal his quality in spite of competition?
- Are reserve prices driven to opportunity costs?

Introduction
Our paper

Questions:

- Can a seller still signal his quality in spite of competition?
- Are reserve prices driven to opportunity costs?

Results:

- Signaling quality requires sacrifice of trade opportunity
- In large market, if the buyer-seller ratio is sufficiently large, any equilibrium has at least one of the following distortions:
  - A positive measure of sellers pool at the bottom
  - The lowest quality seller sets a reserve price strictly higher than opportunity cost

Model
Environment

We analyze the following environment:

- Second-price auction with reserve price
- Each buyer participates in *at most one* auction; no resale
- Higher-quality object is of higher opportunity cost for seller

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Some notation:

- $J = \{1, ..., N\}$: sellers/objects
- $S = [\underline{s}, \overline{s}]$: types of a seller
- $I = \{1, ..., kN\}$: buyers
- $\Theta = [0, 1]$: types of a buyer

- Stage 0: Nature draws
  – $(s_1, ..., s_N)$ drawn i.i.d. from cumulative distribution G
  – $(\theta_1, ..., \theta_{kN})$ drawn i.i.d. from cumulative distribution F
  The type of each agent is private information.

Model
Timing

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- Stage 2: Buyers simultaneously decide on participation
- Stage 3: Buyers bids at selected auction

- If object j is sold to buyer i at price t,
  - Seller j obtains t
  - Buyer i obtains $\alpha(\theta_i) + \beta(s_j) - t$
- If seller j fails to sell his object, he obtains $c(s_j)$
- If buyer i fails to buy any object, he obtains 0

Model
Payoffs

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Equilibrium concept: Symmetric PBE

Consider the participation game where:

- Object j is evaluated by buyer type θ with $v_j(\theta) = \alpha(\theta) + b_j$
- Object j is sold at a second-price auction with reserve price r_j
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Let m_j be defined by

$$v_j(m_j) = r_j$$

– m_j is indifferent to winning object j at reserve price

Suppose $N = 3$. Profile $\mathbf{r}$ induces a profile $\mathbf{m}$ of minimum types...

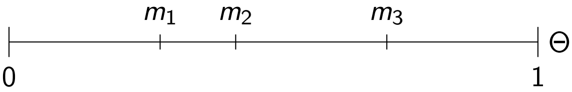

Figure: Buyer behavior

And $\mathbf{m}$ induces a profile of cutoffs $\mathbf{t}$ such that type t_i is indifferent between auctions i and $i-1$

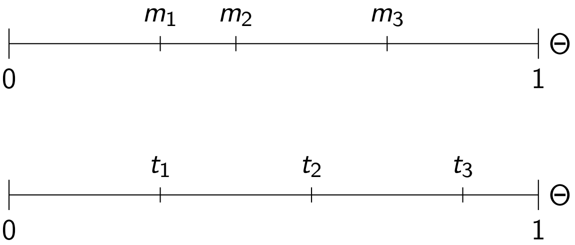

Figure: Buyer behavior

A buyer of type $\theta < t_1$ does not participate

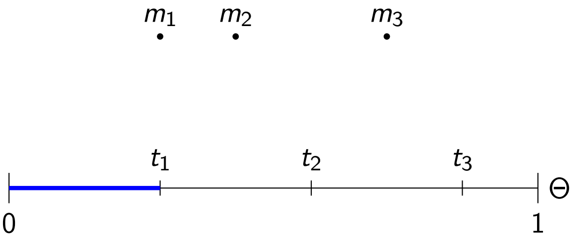

Figure: Buyer behavior

A buyer of type $t_1 \leq \theta \leq t_2$ participates only in auction 1

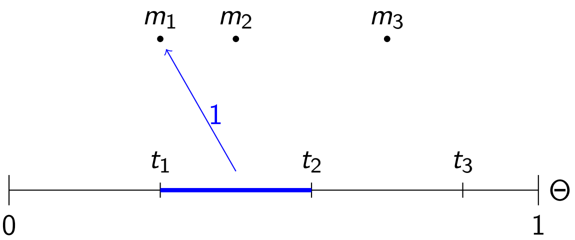

Figure: Buyer behavior

A buyer of type $t_2 \leq \theta \leq t_3$ mixes between auction 1 and 2

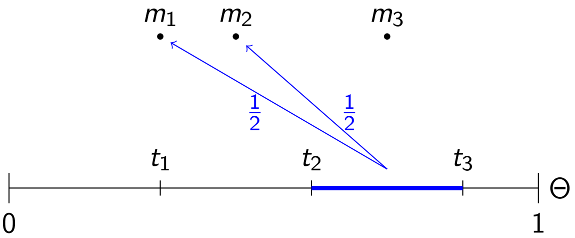

Figure: Buyer behavior

Analysis
Buyer participation

A buyer of type $t_3 \leq \theta \leq 1$ mixes between auction 1, 2 and 3

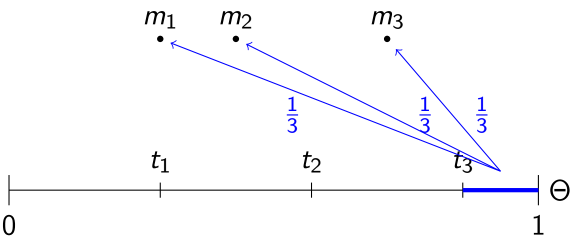

Figure: Buyer behavior

A profile **t** determines **trade opportunity** for each auction

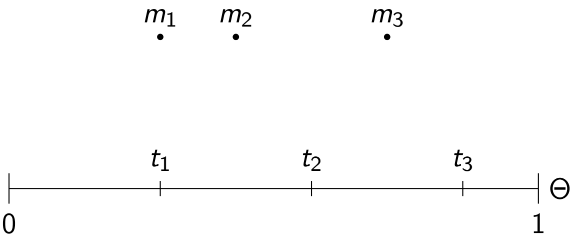

Figure: Buyer behavior

Analysis
Buyer participation

Auction 1: cutoff type attracted is t_3

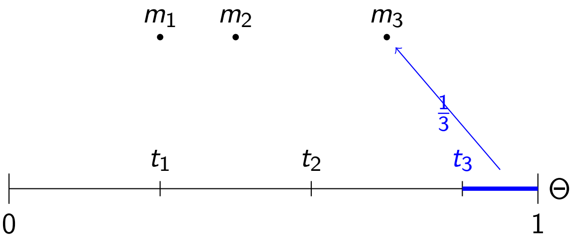

Figure: Buyer behavior

Prob Auction 3 to attract a buyer :

$$p_3 = \frac{1 - F(t_3)}{3}$$

Analysis
Buyer participation

Auction 2: cutoff type attracted is t_2

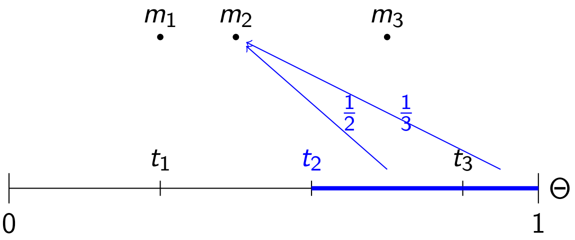

Figure: Buyer behavior

Prob Auction 2 to attract a buyer:

$$p_2 = \frac{F(t_3) - F(t_2)}{2} + \frac{1 - F(t_3)}{3}$$

Auction 3: cutoff type attracted is t_1

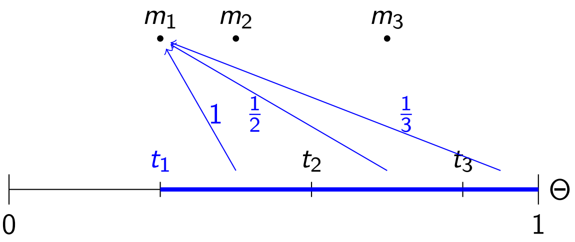

Figure: Buyer behavior

Prob Auction 1 to attract a buyer:

$$p_1 = \frac{F(t_2) - F(t_1)}{1} + \frac{F(t_3) - F(t_2)}{2} + \frac{1 - F(t_3)}{3}$$

Analysis
Buyer participation

- In general

$$p_j = \sum_{j' \geq j}^{N} \frac{F(t_{j'+1}) - F(t_{j'})}{j'}$$

- Expected number of visits kNp_j
- Important term: Probability of no trade

$$Q_j = (1 - p_j)^{kN}$$

Proposition

- $\frac{\partial Q_j}{\partial m_j} > 0$, probability of no trade increases in minimum type.

Seller's problem
Monotonicity

- Each seller chooses reserve price r. Let ρ be seller's strategy
- Each buyer updates beliefs about object quality according $\Lambda(r)$
- Seller's payoff of choosing r can be expressed as:

$$u_s(r;\rho,\Lambda) = c(s)Q(r;\rho,\Lambda) + \Pi(r;\rho,\Lambda)$$

with Q: Probability of no trade, Π: Expected profits

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- How r changes probability of no trade?

$$r = \alpha(m) + \mathbb{E}_{\Lambda(r)}\beta(s)$$

- Quality affects buyer's participation behavior only through induced minimum type
- Quality affects the bids of participating buyers

Proposition

There exists a symmetric PBE.

- In any equilibrium:
  - Higher-quality seller posts a weakly higher reserve price
  - Signaling high quality requires sacrifice of trade opportunity

Equilibrium
Existence and properties

Proposition

There exists a symmetric PBE.

- In any equilibrium:
  - Higher-quality seller posts a weakly higher reserve price
  - Signaling high quality requires sacrifice of trade opportunity
- Problem:
  - Equilibrium could be fully separating or pooling
  - Single crossing conditions difficult to check due to complex strategic interaction

Large game ($N \to \infty$)
Efficiency and Informativeness

We study efficiency and informativeness in a large-market framework

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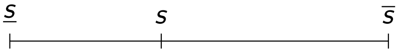

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Efficiency and Informativeness

We study efficiency and informativeness in a large-market framework

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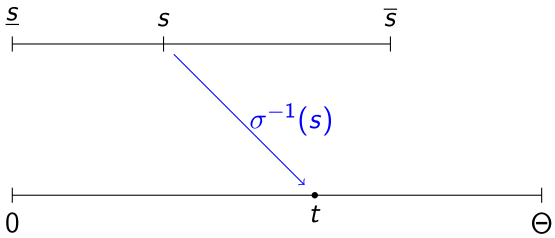
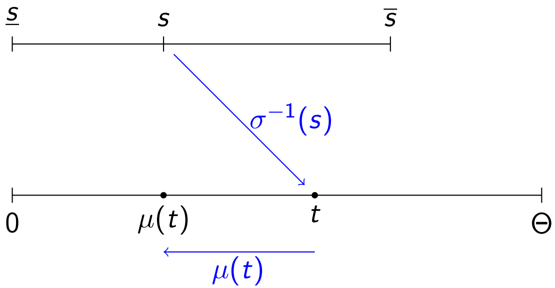
- We match the cutoff buyer type targeted by this auction...



We study efficiency and informativeness in a large-market framework

- Each auction is represented by its object quality
- We match the cutoff buyer type targeted by this auction...
- and the minimum type willing to bid in this auction



Seller's payoffs

Payoff to a type-s seller targeting t is

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$$u_s(t) = c(s)F^1_{\sigma,t}(t)$$

No trade

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Payoff to a type-s seller targeting t is

$$u_s(t) = c(s)F^1_{\sigma,t}(t) + (\alpha(\mu(t)) + \beta(\sigma(t))(F^2_{\sigma,t}(t) - F^1_{\sigma,t}(t))$$

Trade without buyer competition

Seller's payoffs

Payoff to a type-s seller targeting t is

$$\begin{aligned} u_s(t) = {} & c(s)F^1_{\sigma,t}(t) \\ & + (\alpha(\mu(t)) + \beta(\sigma(t))(F^2_{\sigma,t}(t) - F^1_{\sigma,t}(t)) \\ & + \int_t^1 (\alpha(x) + \beta(\sigma(t)))dF^2_{\sigma,t}(x) \end{aligned}$$

Trade with buyer competition

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Trade without buyer competition

Buyer's payoffs

Payoff to a type-θ buyer who visits an auction targeting $t \leq \theta$ is

$$v_\theta(t) = (\alpha(\theta) - \alpha(\mu(t)))F^1_{\sigma,t}(t) + \int_t^\theta (\alpha(\theta) - \alpha(x))dF^1_{\sigma,t}(x)$$

Trade with buyer competition

Equilibrium conditions

Lemma

If (σ, μ) is a separating limit equilibrium, then

$$\beta'(\sigma(t))(1 - F^1(t))\frac{d\sigma}{dt} = (\alpha(\mu(t)) + \beta(\sigma(t)) - c(\sigma(t)))f^1(t)$$

$$\alpha'(\mu(t))F^1(t)\frac{d\mu}{dt} = (\alpha(t) - \alpha(\mu(t)))f^1(t)$$

- Seller's trade-off: Signal higher quality vs Attract less buyers
- Buyer's trade-off: Pay more vs Compete less

Equilibrium conditions

Since F^1 depends on the entire σ, we introduce an auxiliary variable $q(t) = F^1(t)$ and write the previous system as

$$\frac{d\sigma}{dt} = \frac{\alpha(\mu(t)) + \beta(\sigma(t)) - c(\sigma(t))}{\beta'(\sigma(t))} \frac{1}{1-q(t)} \frac{dq}{dt}$$

$$\frac{d\mu}{dt} = \frac{\alpha(t) - \alpha(\mu(t))}{\alpha'(\mu(t))} \frac{1}{q(t)} \frac{dq}{dt}$$

$$\frac{dq}{dt} = k \frac{f(t)}{G(\sigma(t))} q(t).$$

Since $\frac{dq}{dt} > 0$, it is valid to consider q as the "time" variable

Main result: No fully revealing equilibrium

Let $\alpha(t_*) + \beta(\underline{s}) = c(\underline{s})$, and fix $q_0 \in [0, e^{-k(1-F(t_*))}]$.

Theorem

Suppose (σ, μ, t) is a solution to

$$\frac{d\sigma}{dq} = \frac{1}{1-q} \cdot \frac{\alpha(\mu) + \beta(\sigma) - c(\sigma)}{\beta'(\sigma)}$$
$$\frac{d\mu}{dq} = \frac{1}{q} \cdot \frac{\alpha(t) - \alpha(\mu)}{\alpha'(\mu)}$$
$$\frac{dt}{dq} = \frac{1}{q} \cdot \frac{G(\sigma)}{kf(t)}$$

with initial value $(q_0, \underline{s}, t_*, t_*)$. If $k > \frac{g(\underline{s})}{2f(t_*)}$, then for each $q \in [q_0, 1)$, $(\sigma(q), \mu(q), t(q)) = (\underline{s}, t_*, t_*)$.

Proof: unique solution?

Corollaries

In large games, if the buyer-seller ratio is sufficiently large, there is no fully revealing equilibrium with no distortion at the bottom. Either:

- A positive measure of sellers pool at the bottom
- The lowest quality seller sets a reserve price strictly higher than opportunity cost

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The statement is approximately true for finite markets with sufficiently large number of agents
Bad news for efficiency. Either:

- Sellers set reserve price strictly higher than opportunity cost
- Sellers with different (low) quality attract the same number of buyers (in expectation)

Summary

In this paper, we study competing auctions with informed sellers and show that:

- A symmetric PBE exists
- High quality is signaled through sacrifice of trade opportunity
- For large enough number of firms, there is always distortion at the bottom